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ACTIVE RESISTANCE EMULATION IN THREE-PHASE RECTIFIER WITH SUBOPTIMAL CURRENT INJECTION

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Abstract: *In this paper, suboptimal current injection in three-phase diode bridge rectifiers that apply switching resistance emulators is analyzed. Two rectifiers are focused, one that applies filtering of the resistance emulator output current, and the other one that does not. Models that cover both the continuous and the discontinuous conduction mode of the rectifiers are developed. Optimization is performed, showing that both of the rectifiers provide about the same performance when the emulated resistance takes the optimal value. The results are experimentally verified.*

Key Words: *AC–DC power conversion, converters, harmonic distortion, power conversion harmonics, power quality, rectifiers.*

1. INTRODUCTION

In order to reduce total harmonic distortion (THD) of the input currents of three-phase diode bridge rectifiers, current injection methods [1] may be applied. As it is shown in [1], current injection networks require some power to be taken from the rectifier output to provide reduction of the input current THD. There are several methods how the current injection may be implemented, but in all cases the power taken by the current injection network is less than 10% of the rectifier input power. Recovery of this power, i.e. transfer of the power taken by the current injection network to the load, is the topic addressed in this paper. In order to recover the power taken by the current injection network, concept of the loss-free resistor [2] is applied.

The current injection method focused in this paper is suboptimal current injection. The method is discussed in [1], and from the optimal current injection, proposed in [3], differs in the subsystem that provides injection of the harmonics at even triples of the line frequency. This subsystem negligibly improves the input current THD, but increases the system complexity; thus, the rectifiers that apply suboptimal current injection have this subsystem omitted. Typical rectifier of this class is

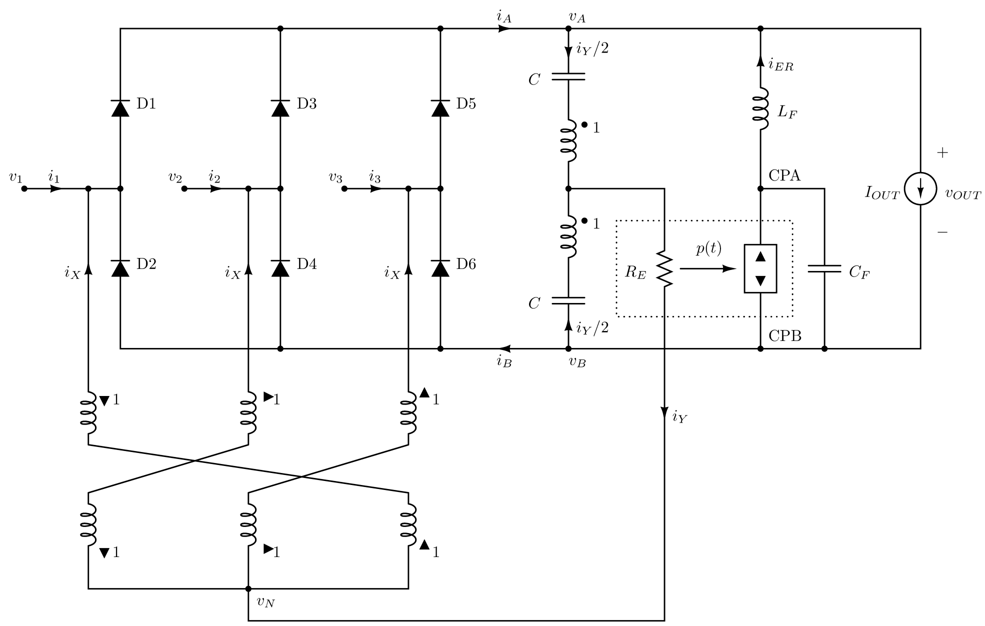

Fig. 1. *Three-phase rectifier with suboptimal current injection and a resistance emulator.*

presented in Fig. 1. The rectifier of Fig. 1 consists of a diode bridge (D1 to D6), a current injection system, and a resistance emulator, encircled with the dotted line, modeled as a loss-free resistor [2]. The resistance emulator might be followed by a filter consisting of L_F and C_F.

Similar approach in the third harmonic current injection based rectifiers is applied in [4], where a passive resistance emulating circuit has been applied. A result of [4] is that filtering of the resistance emulator output current, contrary to initial expectations, does not contribute to the reduction of the input current THD. Instead, the filtering increases the input current THD, thus the filter is omitted from the rectifier proposed in [4]. The reason for this effect is that the output current ripple of the resistance emulator contains a harmonic at sixth multiple of the line frequency with the amplitude close to the amplitude required by the optimal current injection [3].

Effects caused by filtering of the resistance emulator output current in the rectifiers that apply suboptimal current injection are analyzed in this paper. In [5], the same topic is addressed, and feasibility of the concept is experimentally verified. Also, it is observed that the rectifiers with suboptimal current injection that apply the resistance emulator tend to operate very close to the discontinuous conduction mode, since the operating point with the minimum of THD is close to the boundary between the continuous and the discontinuous conduction mode. This paper extends the results of [5] in theoretical direction. The rectifiers are modeled to cover both the continuous and the discontinuous conduction mode, applying techniques of [6] and [7]. Next, an optimization is performed, to determine optimal resistance at the resistance emulator input port. It will be shown that in the case the filtering is applied the optimal operating point is in the continuous conduction mode, while in the case the filtering is not applied the optimal operating point is in the discontinuous conduction mode. Finally, the analytical results are experimentally verified.

2. MODELS

To analayze the rectifier of Fig. 1 in two cases, when complete filtering of the resistance emulator output current is applied, assuming $L_F \to \infty$ and $C_F \to \infty$, and in the case when the filtering is omitted, when $L_F \to 0$ and $C_F \to 0$, appropriate models should be derived. In the analysis, both the continuous and the discontinuous conduction mode are of interest, and the model should cover both of them. The model is derived applying techniques developed in [6] and [7]. Capacitors C of the current injection network are assumed to have negligible voltage ripple, and their voltages are for symmetry reasons assumed as being the same, equal to V_C [7]. To determine the input current waveforms, it is sufficient to determine the waveforms of i_A and i_B (Fig. 1), since $i_Y = i_A - i_B$, $i_X = i_Y/3$, and the conduction of diodes is controlled by the input voltages. It is convenient to determine the waveforms of i_A and i_B from the equivalent circuits of Figs. 2 and 3, developed applying

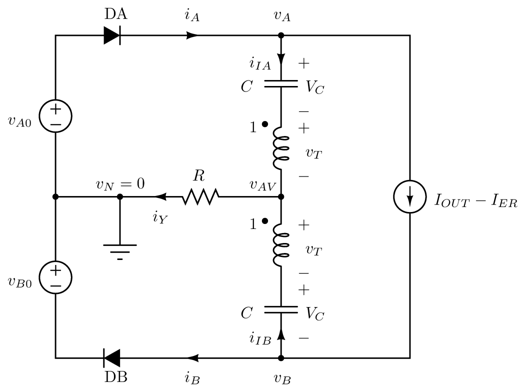

Fig. 2. *Equivalent circuit of the rectifier with filter*.

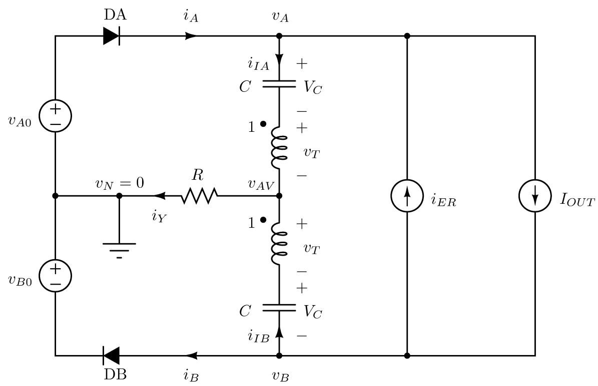

Fig. 3. *Equivalent circuit of the rectifier without filter*.

techniques introduced in [6] and [7] for the analysis of rectifiers in the discontinuous conduction mode. In the analysis of the equivalent circuits, as well as to determine the output voltage, waveforms of v_A and v_B are also of interest.

Let us assume that the supply voltages are

$$v_k = V_m \cos\left(\omega t - (k-1)\frac{2\pi}{3}\right) \quad (1)$$

for $k \in \{1,2,3\}$. Waveforms v_{A0} and v_{B0} of the equivalent circuits of Figs. 2 and 3 are defined as ([6], [7])

$$v_{A0} = \max(v_1, v_2, v_3) \quad (2)$$

and

$$v_{B0} = \min(v_1, v_2, v_3). \quad (3)$$

Let us also define a voltage waveform

$$v_{AV0} = \frac{v_{A0} + v_{B0}}{2}. \quad (4)$$

This waveform plays an important role since under imposed assumptions

$$v_{AV} = \frac{v_A + v_B}{2} \quad (5)$$

regardless conduction of DA and DB.

In the continuous conduction mode, diodes DA and DB of the equivalent circuits of Figs. 2 and 3 conduct during the whole period, resulting in $v_A = v_{A0}$ and $v_B = v_{B0}$. In this case $i_A > 0$ and $i_B > 0$ during the whole period, resulting in $v_{AV} = v_{AV0}$ and $i_Y = v_{AV0}/R$. This applies for both of the analyzed equivalent circuits.

In discontinuous conduction modes, there are intervals of time in which i_A or i_B are equal to zero.

These intervals correspond to diodes DA or DB being reverse biased. Out of four possible combinations of diode states only three are of interest, characterized by at least one diode in conducting state.

2.1. Model of the rectifier with filter

The rectifier with filter is analyzed assuming value of a current defined as $I_Z = I_{OUT} - I_{ER}$. Solving the circuit of Fig. 2, a model that consists of three linear segments is obtained. The first segment covers the continuous conduction interval, where both DA and DB conduct, resulting in $v_A = v_{A0}$ and $v_B = v_{B0}$, $i_Y = v_{AV0}/R$, $i_A = I_Z + i_Y/2$, and $i_B = I_Z - i_Y/2$. This applies for $v_{AV0} > -2RI_Z$ and $v_{AV0} < 2RI_Z$.

In the case $v_{AV0} > 2RI_Z$, DB is off, and $v_A = v_{A0}$, $v_B = -v_{A0} + 4RI_Z$, $i_Y = 2I_Z$, $i_A = 2I_Z$, and $i_B = 0$.

For $v_{AV0} < -2RI_Z$, DA is off, and $v_A = -v_{B0} - 4RI_Z$, $v_B = v_{B0}$, $i_Y = -2I_Z$, $i_A = 0$, and $i_B = 2I_Z$.

After the waveforms are computed, actual output current is computed from $I_{OUT} = I_Z + I_{ER}$, where I_{ER} is obtained as

$$I_{ER} = \overline{\frac{\overline{Ri_Y^2}}{v_A - v_B}} \qquad (6)$$

where overline represents averaging over the line period.

When the waveforms of the rectifier voltages and currents are determined, parameters like the input current THD and the power factor are computed applying standard procedures.

2.2. Model of the rectifier without filter

In the same manner as for the rectifier with filter, in the case the filter is omitted waveforms of the rectifier voltages and currents are obtained solving the equivalent circuit of Fig. 3. For the continuous conduction interval, when both DA and DB conduct, $v_A = v_{A0}$, $v_B = v_{B0}$, and $i_Y = v_{AV0}/R$. In that case

$$i_A = I_{OUT} - \frac{v_{B0}}{v_{A0} - v_{B0}} \frac{v_{AV0}}{R} \qquad (7)$$

and

$$i_B = I_{OUT} - \frac{v_{A0}}{v_{A0} - v_{B0}} \frac{v_{AV0}}{R}. \qquad (8)$$

This applies for $i_A > 0$ and $i_B > 0$.

In the case DB is off, $v_A = v_{A0}$,

$$v_B = -v_{A0} \frac{v_{A0} - 2RI_{OUT}}{v_{A0} + 2RI_{OUT}} \qquad (9)$$

$$i_A = i_Y = 2I_{OUT} \frac{v_{A0}}{v_{A0} + 2RI_{OUT}} \qquad (10)$$

and $i_B = 0$. This applies for $v_B < v_{B0}$.

Similarly, when DA is off

$$v_A = -v_{B0} \frac{v_{B0} + 2RI_{OUT}}{v_{B0} - 2RI_{OUT}} \qquad (11)$$

$v_B = v_{B0}$, $i_A = 0$, and

$$i_B = -i_Y = 2I_{OUT} \frac{v_{B0}}{v_{B0} - 2RI_{OUT}}. \qquad (12)$$

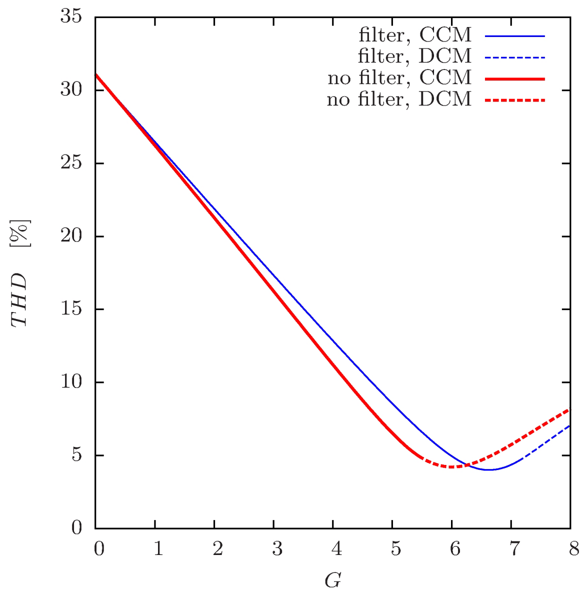

Fig. 4. *Dependence of the input current THD on G.*

This applies for $v_A > v_{A0}$.

3. OPTIMIZATION

After the converter models are derived, numerical computation is applied to determine optimal values for the emulated resistance in both of the considered cases. In order to generalize the results, normalization is performed, and the input current THD is expressed in terms of normalized conductance of the resistance emulator input port, defined as

$$G = \frac{V_m}{RI_{OUT}}. \qquad (13)$$

Numerical computation is performed applying GNU Octave, and obtained dependence of the input current THD on G is presented in Fig. 4, where the full line corresponds to the continuous conduction mode, while the dashed line corresponds to the discontinuous conduction mode.

According to the diagrams of Fig. 4, the result of optimization in the case the filter is applied is that the minimum of the input current THD of $THD_{\min} = 4.01\%$ is obtained for $G = 6.62$ in the continuous conduction mode, requiring the resistance emulator to process 8.66% of the input power. In the case the filtering is omitted, minimum of the input current THD of $THD_{\min} = 4.22\%$ is obtained for $G = 6.50$ in the discontinuous conduction mode, and the resistance emulator processes 8.40% of the input power.

Comparison of the simulation results indicate that both of the rectifiers provide about the same performance. Advantage provided by applying filtering of the resistance emulator output current is slightly better THD, while omitting the filter provides slight reduction of the power processed by the resistance emulator. According to the results, for applications in practice the solution without filtering would be advised, regardless slightly higher THD values.

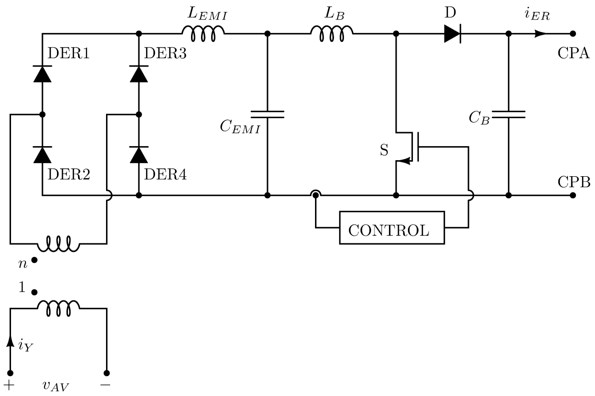

Fig. 5. *Resistance emulator.*

4. EXPERIMENTAL RESULTS

To provide experimental verification of the theoretically obtained results, a rectifier with the rated power of 2 kW is built. The rectifier is intended to operate with the phase voltage amplitude of $V_m = 140\ \text{V}$, at the line frequency of $50\ \text{Hz}$. The resistance emulator is built using a transformer with the turns ratio $1:3.2$, single-phase diode bridge, and a boost converter with current mode control, as depicted in Fig. 5. A filter consisting of $L_{EMI} = 140\ \mu\text{H}$ and $C_{EMI} = 320\ \text{nF}$ is applied in front of the boost converter to reduce the electromagnetic interference. Similarly, the output capacitor of the boost converter, $C_B = 200\ \mu\text{F}$, is applied to absorb spectral components of the diode current at high frequencies, in the order of magnitude of the switching frequency. Inductor of $L_B = 500\ \mu\text{H}$ is applied. If low frequency filtering, at several multiples of the line frequency is needed, the additional filter consisting of $L_F = 1.25\ \text{mH}$ and $C_F = 2200\ \mu\text{F}$, shown in Fig. 1, could be applied. The resistance emulator of Fig. 5 is applied to emulate resistance slightly lower than optimal, since a part of the current injection network optimal resistance is obtained from the current injection network losses. These losses are primarily located in the

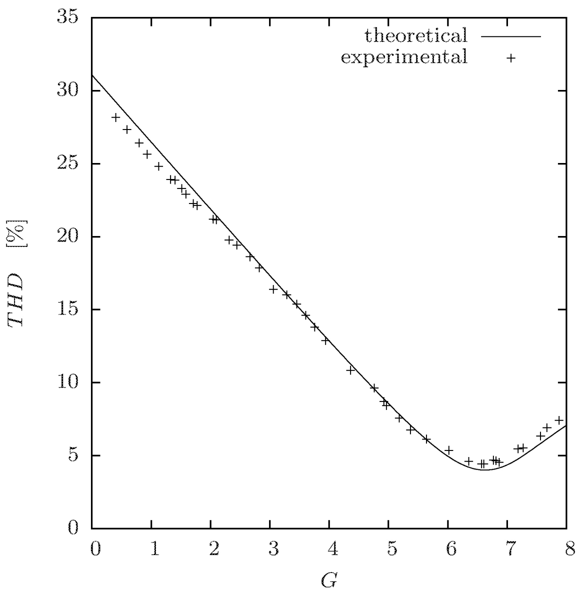

Fig. 6. *Experimental results,* $THD(G)$, *with filter.*

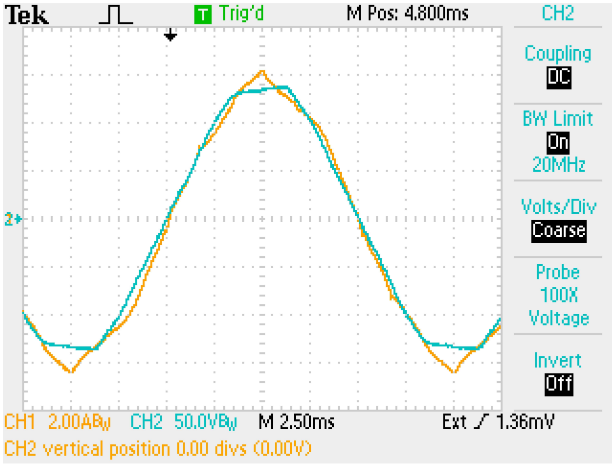

Fig. 7. *Waveforms of* v_1 *and* i_1 *at* $I_{OUT} = 5\ \text{A}$ *and* $THD_{\min}$, *with filter.*

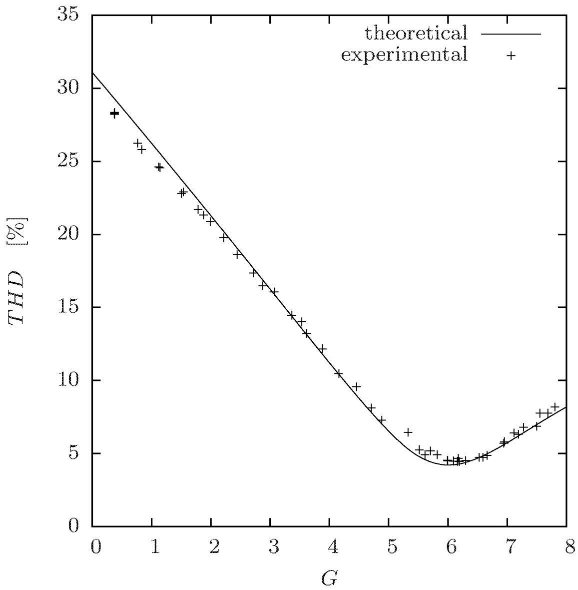

Fig. 8. *Experimental results,* $THD(G)$, *without filter.*

current injection device, being modeled by a series resistance of $R_{CID} = 1.35\ \Omega$.

Experimentally obtained dependence of the input current THD on normalized conductance in the current injection network in the case filtering of the resistance emulator output current is applied is presented in Fig. 6. The diagram is obtained for the rectifier operating at $I_{OUT} = 5\ \text{A}$. In the diagram of Fig. 6, the thin line presents theoretically obtained dependence, already shown in Fig. 4, while the crosses correspond to the experimental data points. The experimental results are in good agreement with the theoretical expectations. In Fig. 7 waveforms of the input voltage and the input current at the first phase of the rectifier, v_1 and i_1, are presented in the case the rectifier is tuned to operate at the minimum of the input current THD.

In the case filtering of the resistance emulator output current is not applied, experimentally obtained dependence of the input current THD on normalized conductance in the current injection network is presented in Fig. 8. Again, the diagram is obtained for the rectifier

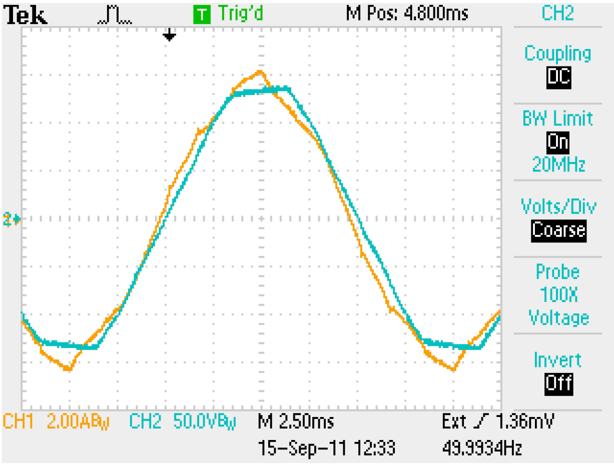

Fig. 9. *Waveforms of* v_1 *and* i_1 *at* $I_{OUT} = 5\ \mathrm{A}$ *and* $THD_{\min}$*, without filter.*

operating at $I_{OUT} = 5\ \mathrm{A}$, and the experimental results are in good agreement with the theoretical predictions. Waveforms of the input voltage and the input current at the first phase of the rectifier for the rectifier operating with the minimum of the input current THD are presented in Fig. 9.

The diagrams of Figs. 7 and 9 are recorded at the output power of about $1150\ \mathrm{W}$, corresponding to the output current $I_{OUT} \approx 5\ \mathrm{A}$ and the output voltage $V_{OUT} \approx 230\ \mathrm{V}$. In both cases, efficiency of the rectifiers is measured to be about 95%.

Comparison of the waveforms of Figs. 7 and 9 yield conclusion that the waveforms are almost the same. According to the analysis performed, as well as the experimental experience, it can be concluded that slight improvement in the performance does not justify application of the resistance emulator output current filter, although the reasons to avoid the filter are not that clear as in [4].

5. CONCLUSIONS

Resistance emulation in three-phase diode bridge rectifiers that apply suboptimal current injection is analyzed in this paper. In comparison to the third harmonic current injection, where passive resistance emulation might be applied [4], suboptimal current injection requires application of switching resistance emulators. Optimal emulated resistance of the resistance emulator and filtering of the resistance emulator output current are discussed. Since the rectifiers with suboptimal current injection tend to operate close to the discontinuous conduction mode, rectifier models that cover both the continuous and the discontinuous conduction modes are developed. The method of equivalent circuits introduced in [6] is applied. The rectifiers with suboptimal current injection are treated as resistive circuits, assuming constant voltages across the capacitors of the current injection network, as well as assuming ideal filtering if the resistance emulator output filter was applied. This significantly simplifies the analysis, making it comparable in complexity to the analysis of multipulse rectifiers presented in [7]. Equivalent circuits are presented for both of the considered rectifiers, the one that applies ideal filtering of the resistance emulator output current, and the one that does not apply filtering. Main equations that describe the models are given.

Obtained rectifier models are applied to perform numerical optimization of the emulated resistance in order to minimize total harmonic distortions of the input currents. For the rectifier that applies filtering, the minimum of $THD_{\min} = 4.01\%$ is obtained in the continuous conduction mode, while for the rectifier that does not apply filtering the minimum of $THD_{\min} = 4.22\%$ is obtained in the discontinuous conduction mode. The rectifier without filtering requires slightly lower power to be processed by the resistance emulator, equal to 8.40% of the input power, in comparison to the rectifier with filter that requires 8.66%. Although application of the filter provides somewhat lower input current THD, simulation results and the experimental experience do not justify application of the resistance emulator output filter.

Analytical results are verified on a rectifier laboratory model with the rated power of $2\ \mathrm{kW}$. The experimental results are in good agreement with the theoretical predictions.

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