Thermal Power Density Barriers of Converter Systems

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Abstract

With the desire to integrate power converters into larger systems, high power density became a very important issue. In this paper we discuss the thermal limits of the power density of converter systems. It is shown that these limits are partly given by today's technology and partly given by physics. Employing the given equations, one can quickly estimate the maximum possible power density of a converter design. Considering the limitations discussed in the paper will be helpful in setting up roadmaps related to future trends in converter power density.

Introduction

We define power density of a converter system as converter output power divided through converter volume. With the general trend to integrate power converters into larger and complex systems, there is a strong pressure on research and development to increase the power density. The power density is limited by the selected topology, the inductive components, the capacitive components and the cooling system (see [1] for a detailed discussion).

In section 1 we discuss the general limits of forced convection cooling which is often much more effective than natural convection. Employing a heat sink plus fan, the cooling system power density is limited by the fan's power consumption, in some applications by the noise of the fan, and by the thermal conductivity of the heat sink material. Optimizing the heat sink geometry for a given fan characteristic minimizes the cooling system volume for a given thermal resistance. In order to be able to generally compare different cooling system technologies (like different power levels, operating temperature levels, natural or forced convection, water cooling, heat pipes, …) we employ the Cooling System Performance Index (CSPI) [2], which can also be understood as a volumetric thermal conductivity [3], defined as

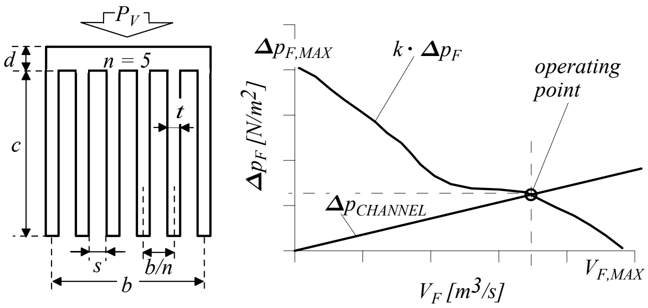
$$CSPI\left[\tfrac{W}{K\cdot liter}\right] = \frac{G_{th,S-a}^{(HS)}\left[\tfrac{W}{K}\right]}{Vol_{CS}\left[liter\right]} = \frac{1}{R_{th,S-a}^{(HS)}\left[\tfrac{K}{W}\right]\cdot Vol_{CS}\left[liter\right]} \qquad (1)$$

In section 2 we investigate how the limits found in section 1 influence the power density of the whole converter system. Simple equations to estimate the power density of cube-shaped converter systems are given. The discussion performed in section 2 will make clear that in case of pure natural convection, small systems will always show higher power densities than large systems. But assembling one large converter from many small sub-converters does not help to increase the total power density, because additional outer surface will be needed for cooling all internal sub-converters. Generally, the absolute dimensions have a strong impact on the power density.

In section 3 we shown that besides optimization of the CSPI, a further and often significant increase of the converter power density can be achieved by increasing the temperature levels of power semiconductors and/or heat sink surface.

1. Cooling System Optimization

In order to optimize a heat sink employing forced convection, one has to take into account the thermal resistance of the heat sink material, the thermal resistance due to convection, and the temperature increase of the air flowing through the heat sink channels. The following optimization is based on the heat sink design of **Fig.1**.



(a) (b)

Fig.1: (a) Geometry of the heat sink with thermal losses P_V *[W]* covering the area $A_{CHIP} = b\cdot L$ with L as heat sink length in air flow direction. **(b)** Fan characteristic showing fan pressure dependent on the air flow V_F.

Generally, the fin geometry defines channels which provide a pressure drop $\Delta p_{CHANNEL}$ for the air flow. The air flow driven by a fan behaves according to the fan characteristic as shown in Fig.1(b). The resulting operating point defines the air flow that, based on the fin geometry, is laminar or turbulent. Based on flow and air properties, the convective heat transfer from the fin surface into the air in the channels can be calculated. All these relations can be described employing analytical and empirical equations ([2], [4]), which allows a systematic optimization of the fin geometry for a given fan. Following the procedure described in [4], the total thermal resistance of the heat sink (from heat sink surface to ambient air at the fan inlet) is

$$R_{th,S-a}^{(HS)} = R^{*}_{th,FIN} + R_{th,conv} + R_{th,\Delta T} \tag{2}$$

with conductive heat flow through the fins

$$R^{*}_{th,FIN} \approx \frac{1}{2\lambda_{HS} L} \cdot \left(1 - 10^{-3} \frac{V_{F,MAX}}{\Delta p_{F,MAX}} \frac{L}{s^2 c^2}\right)^{-1}, \tag{3}$$

convective heat flow from fin surface into air

$$R_{th,conv} \approx \left(1.3 \cdot 10^4 \frac{\Delta p_{F,MAX}}{V_{F,MAX}} \frac{s^4}{L^2}\right) \Bigg/ \left(1 + 2 \cdot 10^3 \sqrt{\Delta p_{F,MAX}} \cdot \frac{s^2}{L}\right) \tag{4}$$

and a contribution due to the temperature rise of the air heating up along the channel from inlet to outlet

$$R_{th,\Delta T} \approx \frac{7.5 \cdot 10^{-4}}{V_{F,MAX}}. \tag{5}$$

Generally, maximum air flow rate $V_{F,MAX}$, pressure drop $\Delta p_{F,MAX}$ and power consumption of the fan P_{FAN} are dependent on the fan's rotating speed N *[rmp]* and diameter D *[m]* as described generally in [5] as

$$V_{F,MAX}\left[m^3/s\right] = k_1 \cdot N \cdot D^3 \tag{6}$$

$$\Delta p_{F,MAX}\left[N/m^2\right] = k_2 \cdot N^2 \cdot D^2 \tag{7}$$

$$P_{FAN}\left[W\right] = k_3 \cdot N^3 \cdot D^5 \tag{8}$$

Investigating a large number of commercially available fans with wide variations of their geometries [6], we calculated the parameters of (6) - (8) to be within the ranges $k_1 = [0.5\cdot10^{-2} .. 10^{-2}]$, $k_2 = [0.5\cdot10^{-3} .. 10^{-3}]$ and $k_3 = [3\cdot10^{-6} .. 70\cdot10^{-6}]$. For comparison of different heat sink designs concerning power density, we employ the "Cooling System Performance Index (CSPI)" ([2], [3]) as defined in (1) which can be expressed independently from a specific fan by employing (6) - (8). This results in

$$CSPI^{-1} = R_{th,S-a}^{(HS)} \cdot Vol_{CS} =$$

$$\frac{\frac{c^2}{2\lambda_{HS}}\left(1+\frac{c^2}{3A_{CHIP}}\right)}{1-\frac{A_1 \cdot A_{CHIP}}{s^2 \cdot \sqrt[3]{\frac{1}{k_3}\cdot P_{FAN}^{\max}} \cdot c^{1/3}}} + \frac{A_3 \cdot \left(1+\frac{c^2}{3A_{CHIP}}\right) s^2}{1+\frac{A_2 \cdot A_{CHIP}}{s^2 \cdot \sqrt[3]{\frac{1}{k_3}\cdot P_{FAN}^{\max}} \cdot c^{1/3}}} + \frac{A_4 \cdot \left(A_{CHIP} + \frac{1}{3}c^2\right)}{\sqrt[3]{\frac{1}{k_3}\cdot P_{FAN}^{\max}} \cdot c^{1/3}} \tag{9}$$

Here, we use the abbreviations $A_1 = 10^{-3}\cdot k_1/k_2$, $A_2 = 5\cdot10^{-4}\cdot k_2^{-0.5}$, $A_3 = 6.5\cdot k_2^{0.5}/k_1$ and $A_4 = 7.5\cdot10^{-4}\cdot k_1^{-1}$. In (9), the fan speed N has been substituted by the maximum acceptable power consumption of the fan P_{FAN}^{max} employing (8). The "cooling system" volume Vol_{CS} in (9) is defined as volume of heat sink plus volume of fan.

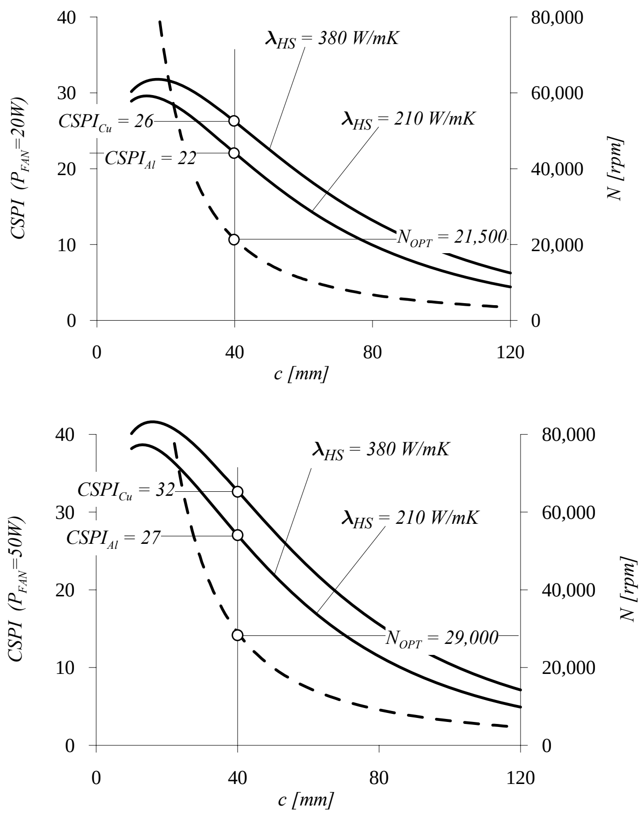


Fig.2: Optimizing a cooling system employing (9) for heat sink materials aluminium and copper. For a typical 5kW-converter with 95% efficiency, we assume the needed area for the power semiconductors to be A_{CHIP}=*32cm²*. The maximum acceptable power consumption of the fan is (a) $P_{FAN,MAX}$=*20W* and (b) $P_{FAN,MAX}$=*50W*.

If the channel width s in (9) is set to values maximizing the CSPI, theoretically optimized curves can be plotted as given in **Fig.2**. If the maximum acceptable power consumption of the fan is set to *20W* (Fig.2(a)) and employing a fan diameter c=*40mm* (equal to fin length as shown in Fig.1(a)), a theoretical maximum $CSPI_{Al}$ *=22* can be achieved for an aluminum heat sink (*210W/Km*), and a maximum $CSPI_{Cu}$ *=26* for copper (*380W/Km*). The required fan speed would be N_{OPT} *= 21,500rpm* for both materials. In case of c=*20mm*, the CSPI-values would be around *30*, but the required fan speed would be *60,000rpm*, significantly increasing noise and reliability problems. In case the maximum acceptable power consumption of the fan is increased to 50W as shown in Fig.2(b), the CSPI for c=*40mm* can be increased to *27* and/or *32*.

Experimental prototypes of optimized heat sinks for typical 5kW-converters are shown in **Fig.3**. These heat sinks have been designed according to the theoretical optimum given in Fig.2(a). Based on measured R_{th}-values, we received $CSPI_{Al}$ *=17.5* and $CSPI_{Cu}$ *=21.6*. Both CSPI-values are 20% below the theoretical optimum because of manufacturing constraints that do not allow to fully exploit the theoretical optimum [7], and the fan employed has a speed of $N=15{,}500rpm$ [6].

(a) (b)

Fig.3: Optimized heat sinks with fan SanAce 40x40x28mm/50dB [6] and with $A_{CHIP}=32cm^2$ (a) aluminum, $n=16$, $s=1.5mm$, $t=1.0mm$, $b=c=40mm$, $d=10mm$, $L=80mm$, $R_{th,S\_a}= 0.26K/W$; (b) copper, $n=23$, $s=1.3mm$, $t=0.5mm$, $b=c=40mm$, $d=10mm$, $L=80mm$, $R_{th,S\_a}=0.21K/W$.

2. Thermal Limits of the Converter Power Density

Generally, the power density of a converter is defined as

$$d_{SYS} = \frac{P_{OUT,SYS}}{Vol_{SYS}} = \frac{\eta_{SYS}}{1-\eta_{SYS}} \frac{\Delta T_{MAX}/R_{th}}{Vol_{SYS}} \tag{10}$$

with converter output power $P_{OUT,SYS}$, converter volume Vol_{SYS}, converter efficiency η_{SYS}, maximum acceptable temperature difference between power semiconductor junction and ambient ΔT_{MAX}, and thermal resistance R_{th} defining the heat flow from the semiconductors to ambient. For the following general discussion we assume the thermal resistance between semiconductors and heat sink surface to be small compared to the thermal resistance associated with convection at the heat sink surface.

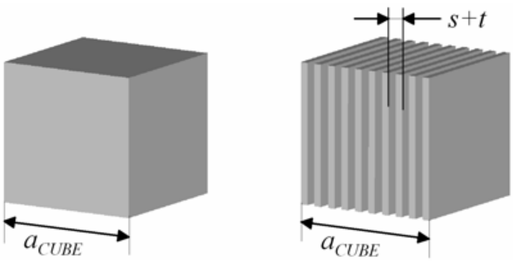


Fig.4: **(a)** Surface area of the cube employing natural convection is proportional to the square of the length $A_{conv} \sim a_{CUBE}^2$. **(b)** 'Internal' surface area consisting of fins is proportional to the third order of the length $A_{conv} \sim a_{CUBE}^2 \cdot a_{CUBE} / (s+t) \sim a_{CUBE}^3$.

As shown in **Fig.4**(a), the natural convection at the outer surface of a cube-shaped converter is proportional to the square of its base length. If forced convection is employed by realizing a part of the converter volume as heat sink plus fan, the effective cooling surface, and, therefore, heat transfer to ambient via forced convection, is proportional to the third order of the length (Fig.4(b)). Here, the fin number $n_{FIN} = a_{CUBE} /(s+t)$ is defined by fin thickness t and channel width s. Because the effective cooling surface is proportional to the volume, it is convenient to use the previously defined CSPI in the following calculations.

The thermal resistance of natural convection at the surface of a cube-shaped (base length a_{CUBE}) converter is

$$R_{th,n,CUBE} = \left(n_{C,f} \;\alpha\; a_{CUBE}^2\right)^{-1} \tag{11}$$

with heat transfer coefficient α $[W/m^2K]$ and number of cooled surfaces $n_{C,f}$. For forced convection, realised by a cooling system integrated in the converter, the according thermal resistance is

$$R_{th,f,CUBE} = \left(k_{CS}\; a_{CUBE}^3\; 10^3\; CSPI\right)^{-1}, \tag{12}$$

which can be directly derived from (1) with $k_{CS} = Vol_{CS} / Vol_{SYS}$ describing the volume share of the cooling system. The thermal resistance in (10) is

$$R_{th} = R_{th,CUBE} = R_{th,n,CUBE} \parallel R_{th,f,CUBE} \tag{13}$$

resulting in a general expression for the converter power density of a cube-shaped converter system as

$$d_{SYS,CUBE}\left[\tfrac{W}{m^3}\right] = \frac{\Delta T_{MAX,S-a}}{\eta_{SYS}^{-1} - 1}\left(n_{C,f}\;\alpha\; a_{CUBE}^{-1} + 10^3\; CSPI \cdot k_{CS}\right) \tag{14}$$

with maximum acceptable sink - ambient temperature difference $\Delta T_{MAX,S-a}$.

To calculate the heat transfer coefficient for natural convection we have to make some assumptions about the heat emitting plate. In (11) – (14) all six cube sides are emitting heat. In the following we will assume that just the horizontal top-plate contributes to natural convection. Employing (3.328) in [8], the Rayleigh number for a heated horizontal square-shaped plate is generally given as

$$Ra_{TOP} = \beta_\infty (T - T_a)\, g\, L^3\, \nu^{-2}\, Pr \tag{15}$$

We set $g=9.81$, $T_a= 20°C$ and characteristic length $L= A/U= 0.25\ a_{CUBE}$, $\beta_\infty= 1/(273+20)=3.4e\text{-}3$. Assuming a plate temperature $T=60°C$ we can set $\nu(60°C)= 2e\text{-}5$ and $Pr(60°C)= 0.7$. This results in a Nusselt number

$$\begin{aligned} Nu_{m,TOP,60°C} &= 0.54\; Ra_{TOP,60°C}^{1/4} = \\ &= 0.54 \cdot (3.66 \cdot 10^7\; a_{CUBE}^3)^{1/4} = 42\; a_{CUBE}^{3/4} \end{aligned} \tag{16}$$

With the definition of the Nusselt number

$$Nu_m = \alpha L / \lambda_{AIR} \tag{17}$$

and the thermal air conductivity $\lambda_{AIR}(60°C) = 30e\text{-}3$ we receive for the heat transfer coefficient of the horizontal top-plate of the cube-shaped volume

$$\alpha_{TOP,60°C} = 5\; a_{CUBE}^{-1/4} \tag{18}$$

In case of vertical walls, the Nusselt number is according to (3.328) in [8]

$$Nu_{m,WALL} = \left[0.825 + 0.387\ Ra^{1/6} \cdot (1 + (0.492\ Pr^{-1})^{9/16})^{-8/27}\right]^2 \tag{19}$$

or, especially for *T=60°C*,

$$Nu_{m,WALL,60°C} = (0.825 + 5.9\ a_{CUBE}^{1/2})^2 \tag{20}$$

resulting in a heat transfer coefficient

$$\alpha_{WALL,60°C} = \frac{1}{12.2\ a_{CUBE}} (1 + 7.2\ a_{CUBE}^{1/2})^2 \tag{21}$$

Assuming natural convection only at the horizontal top plate of the cube-shaped converter, (14) can be combined with (18) giving

$$d_{SYS,CUBE}^{(TOP,60°C)} \left[\tfrac{W}{m^3}\right] = \frac{\Delta T_{MAX,S-a}}{\eta_{SYS}^{-1} - 1} \left(5\ a_{CUBE}^{-5/4} + 10^3\ CSPI \cdot k_{CS}\right) \tag{22}$$

which is plotted in **Fig.5**. As shown there, in case of dominating forced convection, the converter power density d_f is in approximation independent from the absolute converter size, while in case of pure natural convection d_n decreases inverse proportional with the base length. One can also see that in case of pure natural convection, small systems will always show higher power densities than large systems. Assembling one large power converter from many small sub-converters does not change this fundamental relationship between absolute size and power density because additional outer surface and/or channel volume will be needed for cooling all internal sub-converters.

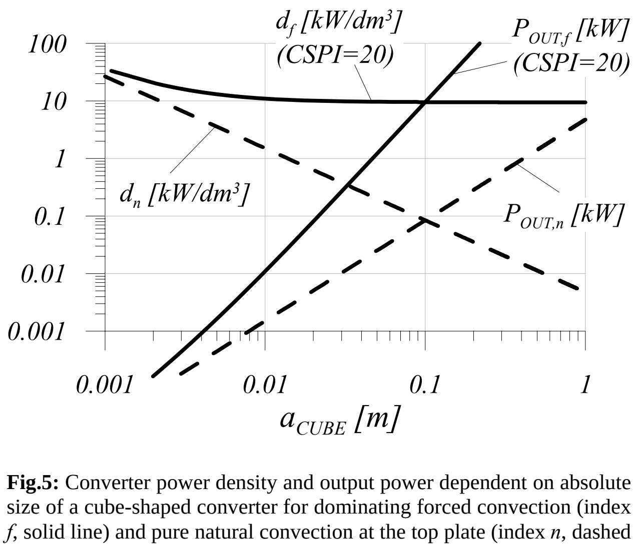


Fig.5: Converter power density and output power dependent on absolute size of a cube-shaped converter for dominating forced convection (index *f*, solid line) and pure natural convection at the top plate (index *n*, dashed line). Parameters are $\eta_{SYS}=0.95$, $\Delta T_{MAX,S-a}=50°C$, $CSPI=20$, $k_{CS}=0.5$.

For two shapes representing naturally cooled converters with equal efficiency, equal maximum temperature and equal volume, the ration of power densities follows as

$$d_{SYS} / d_{SYS,CUBE} = (1 + 2\ k) / (3\ k^{2/3}) \tag{23}$$

under the assumption that all six surfaces are equally cooled by natural convection. In (23), one shape is a cube, and the deviation of the second shape (base length *a*, height *h*) from the cube is characterized by $k = h/a$. As shown in **Fig.6**, a cube represents the worst shape in terms of maximum power density if all six surfaces are cooled equally (solid line). More realistically for typical converter systems, we assume only one single side to be cooled by natural convection which results in the ratio

$$d_{SYS} / d_{SYS,CUBE} = k^{(-2/3)} \tag{24}$$

shown as dashed line in Fig.6. In this case there is no local minimum. The more plate-like the shape is, the higher the theoretically possible power density will be. Therefore, converters of very small output power employing natural convection and being of flat shape will typically show very high power density.

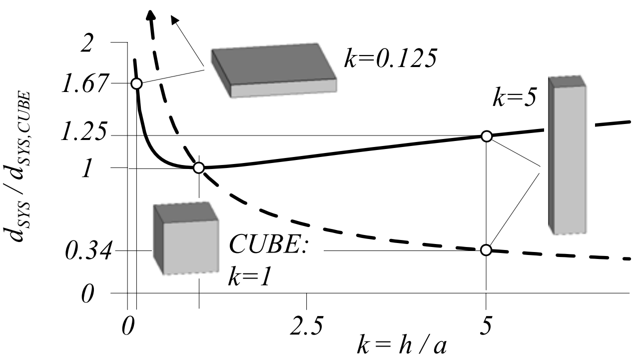


Fig.6: Simple thermal model of converter systems of different shape but with equal volume employing pure natural cooling at their surface. (Solid Line): All surfaces are equally cooled ($n_{CS}=6$). (Slashed Line): Only one surface (square-shaped, base length *a*) is cooled ($n_{CS}=1$).

3. Influence of Power Semiconductor Temperature Levels

The power density of a cooling system can be written as

$$d_{CS} = \frac{P_{OUT,SYS}}{Vol_{CS}} = \frac{\eta_{SYS}}{1 - \eta_{SYS}} \Delta T_{MAX} \cdot CSPI \ . \tag{25}$$

Therefore, besides optimization of the CSPI, a further increase of the power density can be achieved by increasing the maximum acceptable junction temperature. Especially when employing SiC, which allows junction temperatures well above *150°C*, this effect can be exploited [2]. If, at a given temperature $T_{ambient} = 45°C$, the maximum junction temperature can be raised from *125°C* to *175°C* by replacing Si-semiconductors with SiC, the power density d_{CS} will rise by a factor *1.625*.

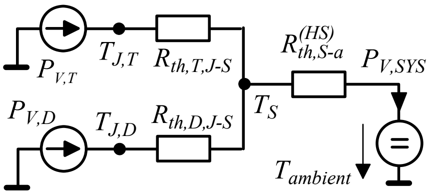


Fig.7: Stationary thermal model of a power transistor (losses $P_{V,T}$) and a diode (losses $P_{V,D}$) mounted onto the heat sink with temperature T_S. The heat sink employing convection is fully represented by $R_{th,S\_a}^{(HS)}$.

Increasing the chip size of a power semiconductor will not only reduce the conduction losses in case of a MOSFET, but another effect valid for all semiconductors is the reduction of the thermal resistance from junction to sink, because $R_{th,J\text{-}S}$ is inverse proportional to the chip size.

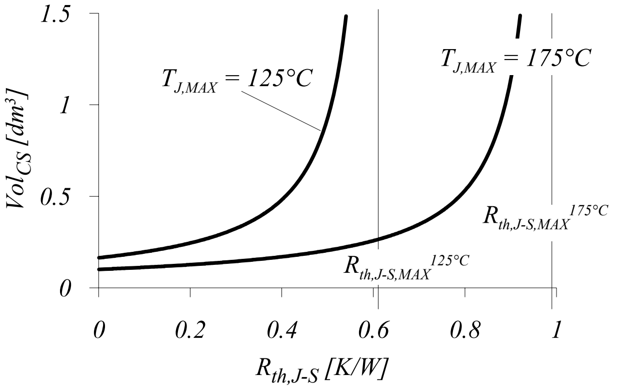


Fig.8: Graphical representation of (26) assuming the thermal equivalent circuit shown in Fig.7 with the parameters η_{SYS} =0.95, T_a=45°C, CSPI= 20, $P_{OUT,SYS}$=5KW.

Assuming equal losses $P_{V,T}$ = $P_{V,D}$ = 0.5 $P_{V,SYS}$ of diode and transistor in **Fig.7**, furthermore setting $R_{th,J\text{-}S}$ = R_{th,T,J_S} = $R_{th,D,J\text{-}S}$, the cooling system volume becomes

$$Vol_{CS}(R_{th,J-S}) = \frac{1}{CSPI}\left(\frac{T_{J,MAX}-T_{ambient}}{P_{OUT,SYS}\left(\eta_{SYS}^{-1}-1\right)} - \tfrac{1}{2}R_{th,J-S}\right)^{-1} \qquad (26)$$

with $T_{J,MAX}$ as the maximum acceptable value for the junction temperatures $T_{J,T}$ and $T_{J,D}$. This fundamental dependency of Vol_{CS} on $R_{th,J\text{-}S}$ is shown in **Fig.8** graphically for two different junction temperatures. As long as $R_{th,J\text{-}S}$ is not too small compared to $R_{th,S\text{-}a}^{(HS)}$, the cooling system volume can be significantly reduced by reducing the thermal resistance of the semiconductors. This can be simply achieved by employing a certain number of power semiconductors in parallel. If the thermal resistance of the semiconductors reaches a critical value $R_{th,J\text{-}S,MAX}$, it becomes impossible to operate the system within the given thermal parameters. Fig.8 also shows that increasing the junction temperatures is very effective in reducing the volume and, therefore, in increasing the power density.

The choice of the thermal resistance values $R_{th,S\text{-}a}^{(HS)}$ and $R_{th,J\text{-}S}$ will define the heat sink temperature T_S. Based on (25), the cooling system volume can be written in dependency of the heat sink temperature as

$$Vol_{CS}(T_S) = \frac{P_{OUT,SYS}\left(\eta_{SYS}^{-1}-1\right)}{CSPI}\left(T_S-T_{ambient}\right)^{-1} \qquad (27)$$

which is shown in **Fig.9** for two different values of the system output power. The closer the heat sink temperature is to the ambient, the larger heat sink volume is needed to keep the converter system within its thermal boundaries. With the heat sink temperature close to the maximum acceptable junction temperature, the volume can be minimized. As already discussed before, this can be achieved by minimizing $R_{th,J\text{-}S}$ (Fig.8).

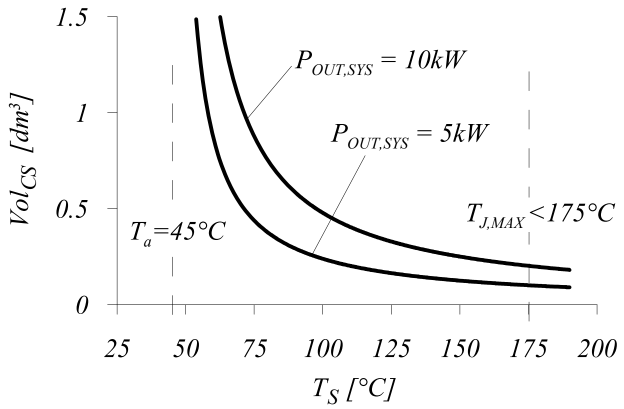


Fig.9: Graphical representation of (27) with the parameters η_{SYS}=0.95, T_a=45°C, CSPI= 20.

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