"Inductive Power Transfer Efficiency Limit of a Flat Half-Filled Disc Coil Pair"

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Problem Statement

What is the theoretical upper bound of the inductive power transfer (IPT) efficiency of a given pair of flat half-filled coils?

- Are there analytic expressions for power transfer and coil losses?
- Are very high switching frequencies (>10MHz) necessary to reach the efficiency limit?

Most Relevant State-Of-The-Art

In order to find the efficiency limit the state-of-the-art within the following topics has been brought together and partially extended:

- Optimum IPT operating conditions[1]
- Coil pair mutual inductance[2]
- Eddy current losses in litz wire[3]
- Radiated losses[4]

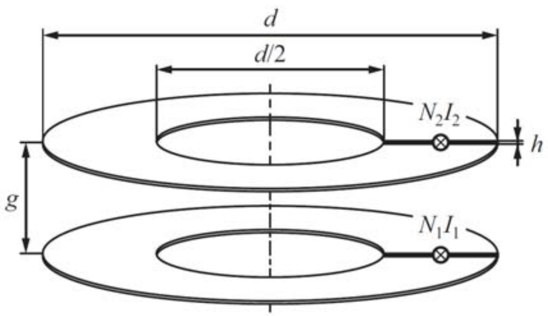
1. R. Bosshard, T. Guillod, and J. W. Kolar, “Electromagnetic field patterns and energy flux of efficiency optimal inductive power transfer systems,” Electr. Eng., vol. 99, no. 3, pp. 969–977, 2016.
2. E. Waffenschmidt and T. Staring, “Limitation of inductive power transfer for consumer applications,” in Proc. 13th Eur. Conf. Power Electron. Appl., 2009, pp. 1–10.
3. M. Leibl, G. Ortiz, and J.W. Kolar, “Design and experimental analysis of a medium-frequency transformer for solid-state transformer applications,” IEEE J. Emerg. Sel. Topics Power Electron., vol. 5, no. 1, pp. 110–123, Mar. 2017.
4. A. Kurs, A. Karalis, R. Moffatt, J. D. Joannopoulos, P. Fisher, and M. Soljačić, “Wireless power transfer via strongly coupled magnetic resonances,” Science, vol. 317, no. 5834, pp. 83–86, 2007

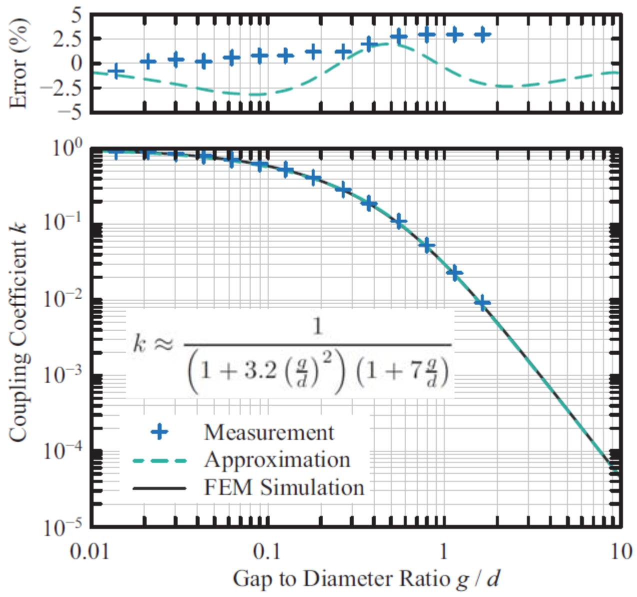
Proposed Approach

- If the currents in primary and secondary coil are 90° phase shifted, and the ratio of their amplitudes is correctly selected ($\frac{I_1}{I_2} \approx \sqrt{\frac{R_2}{R_1}}$) the maximum efficiency $\eta_{12,\text{opt}}$ is obtained. This value is maximized for maximum kQ.

$$\eta_{12,\text{opt}} = \frac{1}{\left(\frac{1}{kQ} + \sqrt{1 + \left(\frac{1}{kQ}\right)^2}\right)^2} \simeq 1 - \frac{2}{kQ}$$

- The coupling coefficient of two flat half-filled disc coils can be approximated with less than 3% error using a FEM-based and experimentally verified curve fit.





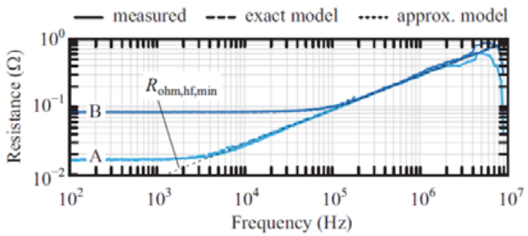
Proposed Approach

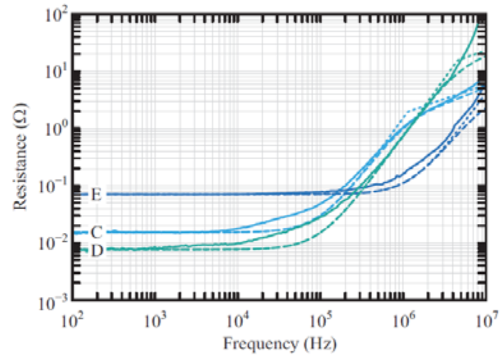
- The coil's ohmic resistances, comprising conduction and eddy current losses, can be asymptotically approximated, as experimentally verified on 5 different coil samples. (skin depth δ, litz wire strand diameter d_r)

$$R_{\mathrm{dc}} = N^2 \frac{3\pi}{k_{\mathrm{cu}} \sigma h}$$

$$\text{if } d_{\mathrm{r}} < 32^{\frac{1}{3}} \delta \qquad R_{\mathrm{ohm,lf}} \simeq R_{\mathrm{dc}} \left(1 + \frac{k_{\mathrm{cu}}^2 h^2 d_{\mathrm{r}}^2}{4\gamma^2 \delta^4}\right)$$

$$\text{if } d_{\mathrm{r}} \geq 32^{\frac{1}{3}} \delta \qquad R_{\mathrm{ohm,hf}} \simeq R_{\mathrm{dc}} \left(\frac{d_{\mathrm{r}}}{4\delta} + \frac{8 k_{\mathrm{cu}}^2 h^2}{\gamma^2 \delta d_{\mathrm{r}}}\right)$$





- Additionally each coil represents a magnetic dipole which radiates power equivalent to the losses in a resistance R_{rad}.

$$R_{\mathrm{rad}} = N^2 \frac{7^2 \pi^5}{3^3 2^5} \sqrt{\frac{\mu_0}{\varepsilon_0}} \left(\frac{df}{c_0}\right)^4$$

Proposed Approach

- With the derived approximations the coil quality factor Q for coils with optimally selected thickness can be calculated as function of the frequency and its maximum can be expressed analytically.

- The quality factor shows a plateau, which is usually exploited in order to minimize the inverter frequency. The efficiency at this plateau depends only on coil diamter d, gap g between the coils, and strand diameter d_r.

$$\eta \approx 1 - \frac{5d_\mathrm{r}}{d}\left(1 + 3.2\left(\frac{g}{d}\right)^2\right)\left(1 + 7\frac{g}{d}\right)$$

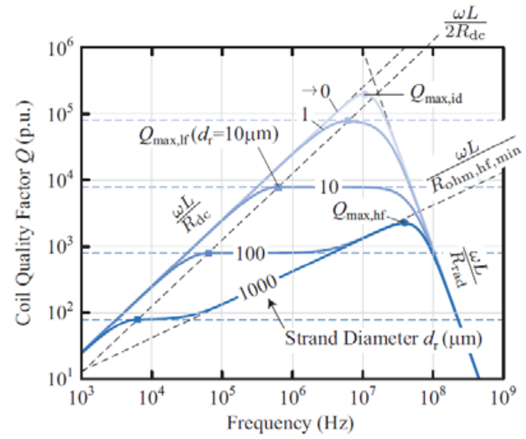
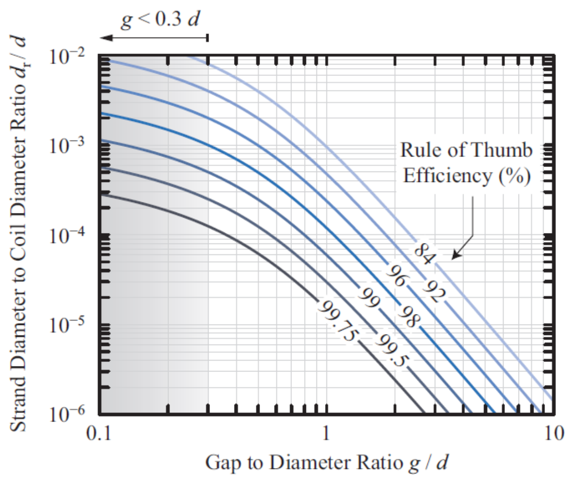


Fig. 10. Quality factor Q of a flat disc coil with diameter $d = 20\,\mathrm{cm}$ as function of the frequency for different strand diameters. The coil height is set to the optimum value but limited to $h < 0.05d$.



Advantages Over Existing Literature

- Simple, yet accurate, analytical approximations for coupling factor, coil AC resistance and efficiency are provided.
- The expressions can be used for dimensioning of simple IPT systems without the need for FEM or to answer the question if the problem at hand can actually be solved by an IPT system.

Constraints, Challenges, Future Steps

- The coils are placed in free space, but in practice either ferrite shielding plates or, with some distance to the coil, conductive shielding plates have to be used. Except for the losses in the ferrite shields, these situations could also be described analytically using current mirroring methods [5].
- In practice the construction of litz wire coils with many thin strands is, which is required for high efficiency, is complicated and usually prevents reaching the efficiency limit.
- Also for low coupling situations the losses in the compensation capacitors limit the overall system efficiency and the tolerances of the compensation capacitors have to be compensated by tuneable capacitors.